Geometry End of the Year Study Guide

Foundations of Geometry

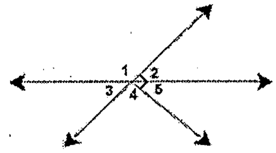
- **Inductive reasoning:** making a conjecture (guess) based on observation of patterns.
- **Deductive reasoning:** proving a statement based on facts (definitions, theorems, postulates, ...)
- **Counterexample:** an example that disproves a statement.
- **Undefined terms:** point, line, plane.
- **Collinear:** on the same line.
- **Coplanar:** in the same plane.
- **Skew lines:** Non-coplanar and never intersect.
- **Postulate:** a statement that is assumed to be true (also called an "axiom").
- **Theorem:** a statement that must be proven true.

Reasoning and Proof

- **Hypothesis:** the "*if*" part of a conditional. (p)
- **Conclusion:** the "*then*" part of a conditional. (q)
- **Conditional statement:** an "if-then" statement.
- **Converse:** switch the "if" and the "then" parts of the conditional.
- **Inverse:** Negate both the "if" and the "then".
- **Contrapositive:** Switch and negate both.
- **Biconditional:** a conditional and its converse are both true and combined into one statement with "if and only if".
- **Counterexample**: a *specific* example where the *hypothesis* of a conditional is true but the *conclusion* is false.

Angle Relationships

- **Angle Bisector:** any figure that divides an angle into two congruent angles.
- **Midpoint of a Segment**: is a point that divides the segment into two congruent segments.
- **Segment Addition Postulate:** If B is between A and C, then AB + BC = AC.
- **Angle Addition Postulate:** If B is in the interior of $\angle AOC$, then $m\angle AOB + m\angle BOC = m\angle AOC$



- **Adjacent angles:** $\angle 3$ and $\angle 4$ (next to)
- **Vertical angles:** $\angle 2$ and $\angle 3$ ($\cong$)
- **Linear pair:** $\angle 1$ and $\angle 3$ (sum of 180)
- **Complementary Angles:** $\angle 2$ and $\angle 5$ (sum of 90)
- **Supplementary Angles:** $\angle 1$ and $\angle 3$ (sum of 180)

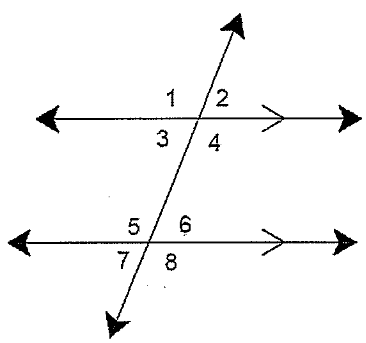
Properties of Equality and Congruence

- **Reflexive Property of Equality:** a = a
- **Symmetric Property of Equality:** If a = b, then b = a.
- **Transitive Property of Equality:** If a = b and b = c, then a = c.
- **Substitution Property of Equality:** If a = b, then a can be substituted for b.

- **Reflexive Property of Congruence:** $\angle A \cong \angle A$
- **Symmetric Property of Equality:** If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.
- **Transitive Property of Equality:** If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$

Parallel and Perpendicular Lines

Parallel Lines: If a transversal intersects parallel lines:



- **Corresponding Angles** are congruent. $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 7 \cong \angle 3$, $\angle 8 \cong \angle 4$
- **Alternate Interior Angles** are congruent. $\angle 3 \cong \angle 6$, $\angle 5 \cong \angle 4$
- **Alternate Exterior Angles** are congruent. $\angle 1 \cong \angle 8$, $\angle 7 \cong \angle 2$
- **Consecutive Interior Angles** are supplementary (sum of 180). $\angle 5 + \angle 3 = 180$, $\angle 6 + \angle 4 = 180$
- **Consecutive Exterior Angles** are supplementary (sum of 180). $\angle 1 + \angle 7 = 180$, $\angle 2 + \angle 8 = 180$

◆ Use Properties of Parallel Lines to prove angle congruence.

◆ Use Converses to prove lines are parallel.

- If two lines are parallel to a third line, then they are parallel to each other.
- In a plane, if two lines are perpendicular to a third line, they are parallel to each other.

Triangle Angle Sum

Triangle angle sum: the angles in a triangle add up to 180 degrees.

Triangle exterior angles: each exterior angle is the sum of the two remote interior angles.

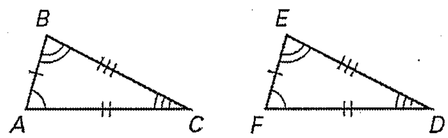
Polygon Angle Sum

Polygon angle sum for a polygon with *n* sides, the angles add up to $(n-2)180$

- Measure of a interior angle of a regular polygon is $\frac{(n-2)180}{n}$
- Sum of the measures of exterior angles is 360.
- Measure of a single exterior angle is $\frac{360}{n}$.

Corresponding Parts

In two congruent figures, all the parts of one figure are congruent to the **corresponding parts** of the other figure.



$\triangle ABC \cong \triangle FED$

Corresponding angles: $\angle A \cong \angle F, \angle B \cong \angle E, \angle C \cong \angle D$

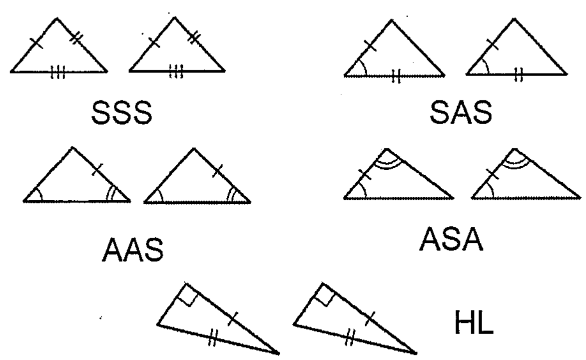
Corresponding sides: $\overline{AB} \cong \overline{FE}, \overline{BC} \cong \overline{ED}, \overline{AC} \cong \overline{FD}$

❖ *When you write a congruence statement always list the corresponding vertices in the same order.*

Congruent Triangles

Third Angle Theorem: If two angles of two triangles are congruent then the third angles are also congruent.

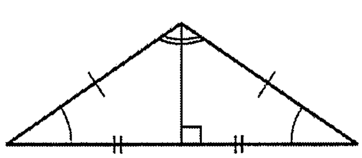
Triangle Congruence Postulates/Theorems:



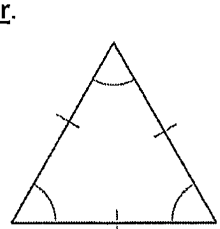
❖ Use **CPCTC** *after* proving triangles are congruent, to prove that parts of the triangles are congruent.

Isosceles and Equilateral Triangles

- **Isosceles Triangle Theorem:** If two sides of a triangle are congruent, then the angles opposite those two sides are congruent.
- In an **isosceles triangle,** the bisector of the vertex angle is the perpendicular bisector of the base.



- If a triangle is **equilateral**, then the triangle is **equiangular**.



Relationships Within Triangles

Point of concurrency: the point where 3 or more lines intersect.

Circumcenter (of a triangle): the point of concurrency of the perpendicular bisectors of a triangle.
- The circumcenter of a triangle is equidistant from the vertices.

Incenter: the point of concurrency of the angle bisectors.
- The incenter of a triangle is equidistant from the sides.

Centroid: the point of concurrency of the medians.
- The centroid is at a point on each median two-thirds of the distance from the vertex to the midpoint of the opposite side.

Orthocenter: the point of concurrency of the altitudes.

Midsegment: the segment that connects the midpoints of a two sides of a triangle.
- The midsegment is ½ the length of the 3rd side and is ǁ to it.

Perpendicular Bisector: If a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints.

Angle Bisector: If a point lies on the angle bisector of an angle, then it is equidistant from the sides of the angle.

Triangle Inequality

- The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- The measure of the third side of a triangle must be less than the sum of the other two sides and greater than their difference.
- Longest side of a triangle is opposite the largest angle.
- Smallest side of a triangle is opposite the smallest angle.

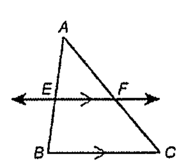
Similarity

Angle-Angle (AA) Similarity Postulate
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

Side-Side-Side (SSS) Similarity Theorem
If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

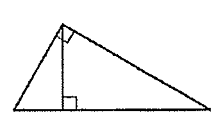
Side-Angle-Side (SAS) Similarity Theorem
If an angle of o ne triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

- If a line parallel to a side of a triangle intersects the other two sides, then it divides those sides proportionally.



$$\frac{AE}{EB} = \frac{AF}{FC}$$

- The altitude to the hypotenuse of a right triangle forms two triangles that are similar to each other and to the original triangle.

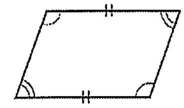


❖ The **geometric mean** of two positive numbers is the positive square root of their product

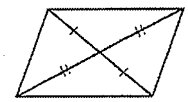
Quadrilaterals

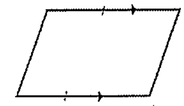
Parallelograms:

- If a quadrilateral is a parallelogram, then its opposite sides and its opposite angles are congruent.



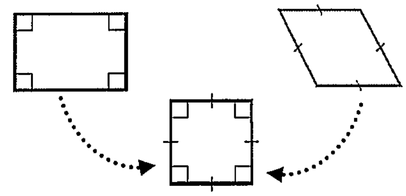
- If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.
- If a quadrilateral is a parallelogram, then its diagonals bisect each other.
- If one pair of opposite sides of a quadrilateral is congruent and parallel, then the quadrilateral is a parallelogram.



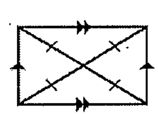


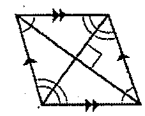
Special Parallelograms:

- A **quadrilateral is a rectangle** if and only if it has four right angles.
- A **quadrilateral is a rhombus** if and only if it has four congruent sides.
- A **quadrilateral is a square** if and only if it is a rhombus and a rectangle.



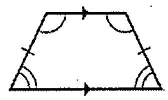
- A **parallelogram is a rectangle** if and only if its diagonals are congruent.
- A **parallelogram is a rhombus** if and only if its diagonals are perpendicular.
- A **parallelogram is a rhombus** if and only if each diagonal bisects a pair of opposite angles.

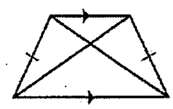




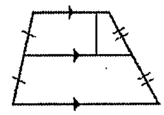
Trapezoids and Kites:

- If a **trapezoid is isosceles**, then each pair of base angles is congruent.
- A **trapezoid is isosceles** if and only if its diagonals are congruent.

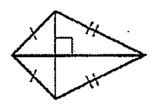


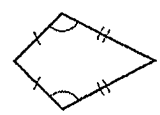


- The **midsegment of a trapezoid** is parallel to each base, and its length is one half the sum of the lengths of the bases.



- If a **quadrilateral is a kite**, then its diagonals are perpendicular.
- If a **quadrilateral is a kite**, then exactly one pair of opposite angles are congruent.





Right Triangle Trigonometry

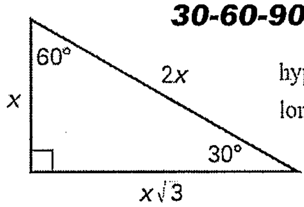
Pythagorean Theorem: $$a^2 + b^2 = c^2$$

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

- Converse of the Pythagorean Theorem: If $c^2 = a^2 + b^2$, then it is a right triangle.
- Acute triangle: If $c^2 < a^2 + b^2$, then it is an acute triangle.
- Obtuse triangle: If $c^2 > a^2 + b^2$, then it is an obtuse triangle.

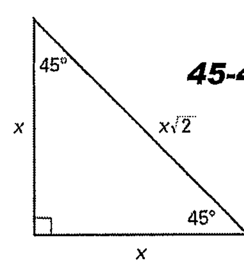
Special right triangles:

30-60-90 Triangle



hypotenuse = 2 • shorter leg
longer leg = shorter leg • $\sqrt{3}$

45-45-90 Triangle

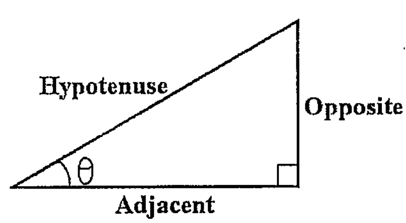


hypotenuse = leg • $\sqrt{2}$

Trigonometry:

Is used to find the lengths of sides in a right triangle when *Pythagorean Theorem* or *Special Right Triangles* won't work.

- **Sine:** $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ **(SOH)**
- **Cosine:** $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ **(CAH)**
- **Tangent:** $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ **(TOA)**



Geometry End of the Year Study Guide

Precision (Number of Significant Figures)

Degree of Precision Rules

- **Leading zeros do not count** *000.01 is precise to 1 digit.*
- **Trailing Zeros before decimal sign do not count.** *40000 is precise to 1 digit*
- **Zeros between 2 nonzeros do count:** *40001 is precise to 5 digits.*
- **Trailing Zeros after decimal sign do count:** *0.3500 is precise to 4 digits.*

Adding or Subtracting –
***Example:* 4.113 + 1000.44 = 1004.553 so round it to 1004.55**
- Multiplication and Division – ***4.01 (3.1) = 12.431 so round it to 12 since 3.1 only has 2 significant digits.***

Simplifying Radicals (for Geometry)

Look for this when using properties of special right triangles.

Factoring out perfect square factors of the radical

Example #1 Simplify $\sqrt{40}$
$\sqrt{40} \rightarrow \sqrt{4} \cdot \sqrt{10} \rightarrow 2\sqrt{10}$

Example #2 Simplify $\sqrt{125}$
$\sqrt{125} \rightarrow \sqrt{25} \cdot \sqrt{5} \rightarrow 5\sqrt{5}$

No Radicals in Denominators!. To get rid of them, rationalize by multiplying by a fraction made up of the radical over itself.

Example #1 Simplify $\frac{5}{\sqrt{3}}$
Multiply $\frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \rightarrow \frac{5\sqrt{3}}{3}$

Example #2 Simplify $\frac{6}{\sqrt{2}}$
Multiply $\frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{6\sqrt{2}}{2} \rightarrow 3\sqrt{2}$

Unit Rates

A unit rate describes how many units of the first type of quantity corresponds to one unit of the second type of quantity. How to Calculate:

Examples: $\frac{12 \text{ in.}}{1 \text{ ft.}}$ and $\frac{1 \text{ ft.}}{12 \text{ in.}}$ $\frac{1 \text{ hr.}}{60 \text{ min.}}$ and $\frac{60 \text{ min.}}{1 \text{ hr.}}$ $\frac{5{,}280 \text{ ft.}}{1 \text{ mi.}}$ and $\frac{5{,}280 \text{ ft.}}{1 \text{ mi.}}$

Unit rates may be used to convert measurements.

Examples: $\frac{60 \text{ mi.}}{\text{hr.}} = \frac{? \text{ ft.}}{\text{sec.}} \rightarrow \frac{60 \cancel{\text{mi.}}}{1 \cancel{\text{hr.}}} \times \frac{5{,}280 \text{ ft.}}{1 \cancel{\text{mi.}}} \times \frac{1 \cancel{\text{hr.}}}{60 \cancel{\text{min.}}} \times \frac{1 \cancel{\text{min.}}}{60 \text{ sec.}} = \frac{88 \text{ ft.}}{\text{sec.}}$

$\frac{88 \text{ ft.}}{\text{sec.}} = \frac{? \text{ mi.}}{\text{hr.}} \rightarrow \frac{88 \cancel{\text{ft.}}}{1 \cancel{\text{sec.}}} \times \frac{1 \text{ mi.}}{5{,}280 \cancel{\text{ft.}}} \times \frac{60 \cancel{\text{sec.}}}{1 \cancel{\text{min.}}} \times \frac{60 \cancel{\text{min.}}}{1 \text{ hr.}} = \frac{60 \text{ min.}}{\text{hr.}}$

Symbols

$\sim$ Similar
$\cong$ Congruent
$\approx$Approximate (use when rounding)
$\perp$ Perpendicular
$\parallel$ Parallel

Coordinate Geometry

Slope formula : $m = \frac{y_2 - y_1}{x_2 - x_1}$

- ***Parallel lines*** have equal slopes.
- ***Perpendicular lines*** have negative reciprocal slopes, and the product of their slopes equals -1. $(m_1 \cdot m_2 = -1)$

Distance Formula

- Distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint Formula

- Midpoint: $m = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

Pythagorean Triples

3, 4, 5
5, 12, 13
7, 24, 25
9, 40, 41

These integers form right triangles.

Converting between Different Units (MC x 3)

Linear Measurements (Perimeters): Use ratio as is.
Square Measurements (Areas): Square the ratio first.
Cubic Measurements (Volumes): Cube the ratio first.

Example: How many feet are there in 2 cubic yards given that there are 3 feet in a yard?

Since this is volume, you need to cube the ratio. In this case the ratio is $\frac{1}{3}$, since $(\frac{1}{3})^3 = \frac{1}{27}$, the ratio between cubic yards and cubic feet is 1 to 27. Therefore there would be 54 cubic feet in two cubic yards.

Geometry End-of-Course Test Map

Reporting Strands	Number of Items MC	CR	SA	Total	Total Number of Points
Logical arguments and proof*	5-8	0	0-1	6-8	6-8
Proving and applying properties of 2-dimensional figures *	15-19	2-4	1-3	21-24	24-26
Figures in a coordinate plane and measurement *	5-8	1-3	0-1	7-9	7-9
Total Number of Items used to determine scale score**	29	5	3	37	
Total Number of Points used to determine scale score**	29	5	6		40
Course-Specific content***	3-5	1-3	0	6	6
Total Number of Items				43	

(there will be 6 try out questions).

Solving "4:5:9" Ratio Type Problems

Example: ***Three angles in a triangle have the ratio 4:5:9. What is the measure of each angle?***

Method #1: **Add up the numbers and use as a denominator.**

In the example, the 3 numbers have a sum of 18.
Use the 18 as a denominator and each number as the numerator. Multiply by the total to get each number.

Small Angle: is $\frac{4}{18}$ of 180 or $\frac{4}{18}(180) = 40°$
Medium Angle: is $\frac{5}{18}$ of 180 or $\frac{5}{18}(180) = 50°$
Largest Angle: is $\frac{9}{18}$ of 180 or $\frac{9}{18}(180) = 90°$

Method #2: **Put an *x* behind each number and create an equation equal to their sum to find common scale factor.**
in this example it would be 4x + 5x + 9x = 180 since the 3 angles in a triangle add up to 180. Then solve for x and get 18x = 180, then x = 10. Then multiple each number in the ratio by the common scale factor to get the numbers.